

Regression models and panel data

Part IV: Statistical tests for panel data models

Rodolfo Metulini

✉ rmetulini@unisa.it

Department of Economics and Statistics (DISES) - University of Salerno

Outline

- 1 Individual and/or time effects
- 2 Correlated effects
- 3 Poolability
- 4 References

Tests on individual and/or time effects

in order to test whether either individual or time effects are present, two approaches are possible:

- ① to start from estimating the model which wipes out the effects (e.g. the within) and then a restricted model (e.g. pooled OLS)
 - ② to start from the pooled OLS and to infer about the presence of such effects analyzing the OLS estimated residuals
- the F test is based on the first approach, and it tests the presence of fixed effects
 - the Breusch-Pagan test is based on the second approach and it is devoted to test the presence of random effects

F test for fixed effects: one way

- One could test the joint significance of these dummies, i.e. $H_0 : \mu_1 = \mu_2 = \dots = \mu_{N-1} = 0$, by performing an F-test.
- Under H_0 the statistic:

$$\frac{\hat{e}_{OLS}^T \hat{e}_{OLS} - \hat{e}_{within}^T W \hat{e}_{within}}{\hat{e}_{within}^T W \hat{e}_{within}} \frac{NT - K - N + 1}{N - 1}$$

follows a Fisher F with $N - 1$ and $NT - K - N + 1$ degrees of freedom.

- the sum of squared residuals of the within model is $\hat{e}_{within}^T W \hat{e}_{within}$ and $NT - K - N + 1$ are the degrees of freedom
- the sum of squared residuals of the OLS model is $\hat{e}_{OLS}^T \hat{e}_{OLS}$ and $NT - K - 1$ are the degrees of freedom
- $N - 1$ is the difference between the two degrees of freedom

F test for fixed effects: two way

- One could test the joint significance of these dummies, i.e.
 $H_0 : \mu_1 = \mu_2 = \dots = \mu_{N-1} = 0, \lambda_1 = \lambda_2 = \dots = \lambda_{T-1} = 0$, by
performing an F-test.
- Under H_0 the statistic:

$$\frac{\hat{e}_{OLS}^T \hat{e}_{OLS} - \hat{e}_{2W,within}^T W \hat{e}_{2W,within}}{\hat{e}_{2W,within}^T W \hat{e}_{2W,within}} \frac{NT - K - N - T + 1}{N + T - 1}$$

follows a Fisher F with $N + T - 1$ and $NT - K - N - T + 1$ degrees of freedom.

- the sum of squared residuals of the two way within model is $\hat{e}_{2W,within}^T W \hat{e}_{2W,within}$ and $NT - K - N - T + 1$ are the degrees of freedom

F test for fixed effects: conditional

Individual
and/or time
effects

Correlated
effects

Poolability

References

- One could test the significance of the time dummies, conditional to the presence of individual dummies, i.e.

$H_0 : \lambda_1 = \lambda_2 = \dots = \lambda_{N-1} = 0$ with $\mu_i \neq 0, i = 1, \dots, T - 1$, by performing an F-test.

- Under H_0 the statistic:

$$\frac{\hat{e}_{within}^T W \hat{e}_{within} - \hat{e}_{2W,within}^T W \hat{e}_{2W,within}}{\hat{e}_{2W,within}^T W \hat{e}_{2W,within}} \frac{NT - K - N - T + 1}{T - 1}$$

follows a Fisher F with $T - 1$ and $NT - K - N - T + 1$ degrees of freedom.

Breusch-Pagan test (i)

- Breusch-Pagan (1980) test is a Lagrange multipliers test based on the ols residuals
- It considers the score vector $g(\theta) = \frac{\delta \ln L}{\delta \theta}$ (i.e. the vector of the partial derivatives of the log-likelihood function from the restricted model, $\sigma_\mu^2 = 0$)
- The variance of the score vector is the information matrix $I(\theta) = E(-\frac{\delta^2 \ln L}{\delta \theta \delta \theta^T})(\theta)$
- Under the H_0 : "OLS is the true model", $g(\hat{\theta}) \sim N(0, V(\hat{\theta}))$
- by denoting with \hat{g} and \hat{V} the score and the estimated variance in the restricted model, $\hat{g}^T \hat{V}^{-1} \hat{g} \sim \chi_n^2$, where n is the number of restrictions.

Breusch-Pagan test (ii)

- To test for the absence of individual (random) effect we impose the null $H_0 : \sigma_\mu^2 = 0$
- The estimator for the parameters $\hat{\theta}$ is the OLS and $\hat{e}_{OLS}^T \hat{e}_{OLS} / NT$ is the estimator for $\hat{\sigma}_v^2$ (since $\sigma_\mu^2 = 0$)
- The statistic $LM_\mu = \frac{NT}{2(T-1)} \left(T \frac{\hat{e}_{OLS}^T B_\mu \hat{e}_{OLS}}{\hat{e}_{OLS}^T \hat{e}_{OLS}} - 1 \right)^2$ asymptotically distributes as a χ^2 with 1 degree of freedom
- To reject (or not) the null hypothesis, we compare the LM_μ terms with the one on χ^2 table.

Breusch-Pagan test (iii)

- Likewise, the test for the absence of time (random) effect impose the null $H_0 : \sigma_\lambda^2 = 0$. The estimator for the parameters ($\hat{\theta}$) is the OLS and $\hat{e}_{OLS}^T \hat{e}_{OLS} / NT$ is the estimator for $\hat{\sigma}_v^2$ (since $\sigma_\lambda^2 = 0$)
- The statistic $LM_\lambda = \frac{NT}{2(N-1)} (N \frac{\hat{e}_{OLS}^T B_\lambda \hat{e}_{OLS}}{\hat{e}_{OLS}^T \hat{e}_{OLS}} - 1)^2$ asymptotically distributes as a χ^2 with 1 degree of freedom.
- The BP test extends to the two-ways error component model and the related statistic is computed as the sum of the previous statistics: $LM_{\mu\lambda} = LM_\mu + LM_\lambda$ and distributes as a χ^2 with 2 degrees, under the null that both individual and time (random) effects are absent.

Application

Application in R: example 4.1 (Croissant, Millo)

Tests for correlated effects

- Let assume $E(X^T v) = 0$
- A critical assumption in error component regression model is that $E(X^T \mu) = 0$
- If the model errors are not correlated with the explanatory variables, both fixed effects (within) and random effects (GLS) estimators are consistent:
 - ① if $E(X^T \mu) = 0$: both estimators are consistent, but random effects is more efficient than fixed effects
 - ② if $E(X^T \mu) \neq 0$: the fixed effect is consistent. The random effects is inconsistent (violation of the basic assumptions)

The Hausman test

- Given π a quantity measuring the relation between X and μ , the Hausman test (1978) is based on testing null hypothesis $H_0 : \pi = 0$ (i.e., uncorrelation)
- We have that $\hat{\pi}^T \hat{V}(\hat{\pi})^{-1} \hat{\pi}$ is distributed as a χ^2 with K degrees of freedom, where $\hat{\pi} = \hat{\beta}_A - \hat{\beta}_B$ and $V(\hat{\pi}) = V(\beta_A) + V(\beta_B)$
- Principle: compare models A and B, where:
 - under H_0 : A and B are both consistent, but B more efficient than A
 - under H_1 : only A is consistent
- A: within, B: GLS. $\hat{q} = \hat{\beta}_{within} - \hat{\beta}_{GLS}$.
If no correlation, $plim \hat{q} = 0$
- the test $\hat{q}^T V(\hat{q})^{-1} \hat{q}$, under H_0 , is distributed as a χ^2 with K degrees of freedom

Application

Application in R: example 4.2 (Croissant, Millo)

Poolability

- The question of whether to pool the data or not naturally arises with panel data.
- The restricted model is the pooled model given by OLS representing a behavioural equation with the same parameters over time and across individuals (just one β , constant along all the sample observations).
- The unrestricted model, however, is the same behavioural equation but with different parameters across individuals (or across time).

Poolability: examples

- Balestra and Nerlove (1966) considered a dynamic demand equation for natural gas across 36 states over 6 years. The question is whether the parameters of this demand equation vary from one year to the other.
- Baltagi and Griffin (1983) considered panel data on motor gasoline demand for 18 OECD countries. They were interested in testing whether the behavioral relationship predicting demand is the same across OECD countries
- Holly et al. (2010) considered whether the effect of income on house prices in 49 US states using a $T=28$ years' panel is varying along states

Chow test - poolability on individuals

- the null hypothesis H_0 is that all β 's are constant along individuals.
- unrestricted estimation consists in estimating (by OLS) one different model for each individual (a total of N models)
- the sum of squares of the residuals for the N (unrestricted) models (stacked) is $\hat{e}_N^T \hat{e}_N$. The degrees of freedom are $NT - NK - N$, because, totally, $NK + N$ parameters need to be estimated.
- The restricted model can be either the OLS ($\hat{e}_{OLS}^T \hat{e}_{OLS}$, $NT - K - 1$) or the within ($\hat{e}_{within}^T \hat{e}_{within}$, $NT - N - K$)
- The statistic (Chow, 1960) is distributed as a F with $(N - 1)K$ and $N(T - K - 1)$ degrees of freedom:

$$\frac{\hat{e}_{within}^T \hat{e}_{within} - \hat{e}_N^T \hat{e}_N}{\hat{e}_N^T \hat{e}_N} \frac{N(T - K - 1)}{(N - 1)K}$$

Application

Application in R: example 8.4 (Croissant, Millo)

References

- Balestra, P., & Nerlove, M. (1966). Pooling cross section and time series data in the estimation of a dynamic model: The demand for natural gas. *Econometrica: Journal of the econometric society*, 585-612.
- Baltagi, B. H., & Griffin, J. M. (1983). Gasoline demand in the OECD: An application of pooling and testing procedures. *European Economic Review*, 22(2), 117-137.
- Breusch, T. S., & Pagan, A. R. (1980). The Lagrange multiplier test and its applications to model specification in econometrics. *The review of economic studies*, 47(1), 239-253.
- Chow, G. C. (1960). Tests of equality between sets of coefficients in two linear regressions. *Econometrica: Journal of the Econometric Society*, 591-605.
- Feng, Q., & Horrace, W. C. (2012). Alternative technical efficiency measures: skew, bias and scale. *Journal of Applied Econometrics*, 27(2), 253-268.
- Hausman, J. A. (1978). Specification tests in econometrics. *Econometrica: Journal of the econometric society*, 1251-1271.
- Holly, S., Pesaran, M. H., & Yamagata, T. (2010). A spatio-temporal model of house prices in the USA. *Journal of Econometrics*, 158(1), 160-173.